Performance of Time-domain Channel Response Based Noise Power Estimation for OFDM

Junhyuk Lee, Hichan Moon
Hanyang University
tobukcho@hanyang.ac.kr, hcmoon@hanyang.ac.kr

Abstract

In many Orthogonal Frequency Division Multiplexing (*OFDM*)*systems, pilots are periodically transmitted for channel estimation. In this paper, we investigate a simple noise power estimation algorithm based on the time-domain channel response computed from the received pilots. The noise power level is estimated by averaging the received power over a predetermined window in the time-domain channel response.By analysis and simulation, the performance of the noise power estimation algorithm is evaluated.*

1. Introduction

Signals-to-noise ratio (SNR) estimation is very important for wireless OFDM systems to improve system performance. It is used for many proposes including adaptive coding and modulation, soft decoding and handoff.

There are many algorithms to estimate SNR of OFDM systems without using pilot signals. Maximum Likelihood (ML) and Minimum Mean Square Error (MMSE) algorithms were presented in [1]. A moment-based noise estimation algorithmwas proposed in [2]. An average SNR estimation algorithm was proposed using cyclic-prefix (CP) of an OFDM system [3]. However, itis not possible to estimate instantaneous SNR with the CP-based algorithm.

However, in many practical OFDM systems, pilots are periodically transmitted for coherent demodulation [4]. It is more reliable to use pilot signals for noise estimation than the algorithms not using pilots, since the pilots are known to both a transmitter and a receiver. Therefore, there have many algorithms to estimate the noise power level using pilot signals [5]-[8].

A subspace-based method was presented to estimate the SNR of the received OFDM symbols [5]. However, this algorithm has disadvantage of high computational complexity because of the operations of correlation and eigenvector decomposition.

In [6]-[8], noise power estimation algorithms have beenproposed using the channel estimation values computed from the pilot signals. These algorithmsrequire channel estimation, which is computationally more complex than noise power estimation. Furthermore, it is difficult to predict the performances, since these algorithms are highly dependent on the accuracy of channel estimation.

Another possible noise power estimation approach is to use the time-domain channel response computed from the received pilots. This algorithm can be implemented with low complexity and provides reasonable performance. However, there has been almost no research on the performance of this scheme. In this paper, we investigate the noise power estimation algorithm based on the time-domain response obtained from the pilots. The performance is evaluated for the noise power estimation algorithm by analysis and simulation.

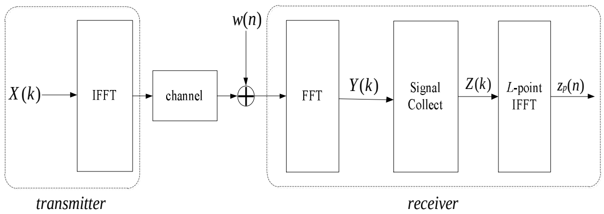

Figure1. System model

2. Noise Power Estimation

Figure1shows a system model for an OFDM system with N subcarriers. The transmitter sends both pilot and data signals. It is assumed that the pilot signals are periodically located in both time and frequency domains as in a LTE (Long Term Evolution) system [4]. In this paper, the noise power of an OFDM symbol is estimated in a frequency domain first and then the estimated value is filtered in a time domain. Therefore, without loss of generality, we focus on an OFDM symbol containing the pilot signals and it is assumed that the noise power is estimated using the OFDM symbol.

It is assumed that the pilot signals are located at $k = mN_P$ for some integer m and that N is a multiple of N_P. Therefore, N_P is the period of the pilot signal in a frequency domain and there are total K

($=N/N_p$)pilot signals in an OFDM symbol. $X(k)$ denotes k -th ($k=0,1,\ldots,N-1$) subcarrier of an OFDM symbol in the frequency domain and can be expressed as

$$X(k)=\begin{cases}\sqrt{P_1}, & \text{if } k \bmod N_p = 0\\ S(k), & \text{otherwise,}\end{cases} \quad (1)$$

where $\sqrt{P_1}$ is sent for the pilot signal and $S(k)$ is the data signal for the k -th subcarrier. Here, $\sqrt{P_1}$ is the power of the pilot signal. $X(k)$ istransformed to a time-domain signal using an Inverse-Fast Fourier Transform (IFFT) block. Then, the time-domain signal is transmitted over a channel with channel response $h(n)$. AWGN $w(n)$ is added to the signal.

In a receiver, the received signal is transformed to a frequency-domain signal using Fast Fourier Transform (FFT) block. Then, the received signal $Y(K)$ in the k -th subcarrieris expressed as

$$Y(k)=\sqrt{P_1}H(k)X(k)+W(k), \quad (2)$$

where $H(K)$ is the channelfrequency response and $W(K)$ is independent and identically distributed complex AWGN. The complex AWGN can be expressed as $W(K)=W_I(k)+jW_Q(k)$, where $W_I(K)$ and $W_Q(K)$ are in-phase and quadrature components. Each component is AWGN with variance $\sigma^2/2$, respectively.

From the received frequency-domain signal, only pilot signals are collected for noise power estimation. The collected pilot signals $Z(k)$ can beexpressedas

$$Z(k)=\begin{cases}Y(kN_p), & \text{if } 0\le k<K\\ 0, & \text{otherwise.}\end{cases} \quad (3)$$

With the proposed algorithm, a time-domain response is obtained from $Z(k)$. The noise power is obtained by averaging the received power of the time domain signal for a time window having small pilot signal power. $Z(k)$ can be represented as

$$Z(k)=\sqrt{P_1}H_d(k)+W_d(k), \quad (4)$$

where $H_d(k)=H(kN_p)$ and $W_d(k)=W(kN_p)$. The $Z(k)$ is contained the pilots and the noise term, respectively.

To estimate the noise power, a time-domain response is obtained for the collected pilot signal $Z(k)$ using an L -point IFFT process of the collected signal, where $L=2^v$ for some positive integers v and $K\le L$. $z_p(n)$ denotes the n -th time component (n=0, 1, 2, …, L-1) of the time-domain signal obtained by the IFFT process from the collected signal. $z_p(n)$ is represented as

$$\begin{aligned}z_p(n) &= IFFT\{\sqrt{P_1}H_d(k)+W_d(k)\}\\ &=\sqrt{P_1}\,h_p(n)+w_p(n),\end{aligned} \quad (5)$$

where $h_p(n)$ is the time response for the channel frequency response and $w_p(n)$ is the residual AWGN in the time domain.

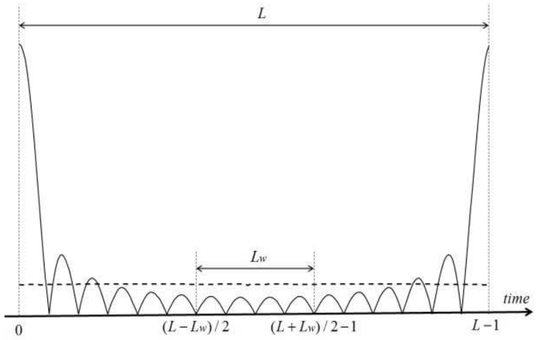

Figure 2. An example ofpower distribution

Figure 2.shows an example of the power distributions of two components of $z_p(n)$. In the figure, the dotted and solid lines show the power of $w_p(n)$ and the power of $h_p(n)$, respectively. Itcan be observed that most power of $h_p(n)$ is distributed near $n=0$ and $L-1$,butthe power of $w_p(n)$ is evenly distributed over all time indices.

The noise power is obtained by averaging the power of $z_p(n)$ over a time window size L_w near center of time index $L/2$. The estimated noise power $\hat{P}_{noise}$ is computed as

$$\begin{aligned}\hat{P}_{noise} &= \frac{L^2}{K}\left(\frac{1}{L_w}\sum_{n=(L-L_w)/2}^{(L+L_w)/2-1}\left|z_p(n)\right|^2\right)\\ &=\frac{L^2}{K}\left(\frac{1}{L_w}\sum_{n=(L-L_w)/2}^{(L+L_w)/2-1}\left|\sqrt{P_1}h_p(n)+w_p(n)\right|^2\right).\end{aligned} \quad (6)$$

Here, the value averaged the power of $z_p(n)$ is scaled by L^2/K to obtain the noise power of each subcarrier in a frequency domain.

3. Performance Analysis

The noise power of an OFDM symbol can be estimated by previous section. If the noise is estimated in an AWGN channel with channel response $h(n)=\delta(n)$, then

$$
\begin{aligned}
\sqrt{P_1}h_p(n) &= \frac{\sqrt{P_1}}{L}\sum_{k=0}^{K-1} H_d(k)e^{j2\pi nk/L} \\
&= \frac{\sqrt{P_1}}{L}\frac{\sin(K\pi n/L)}{\sin(\pi n/L)}e^{j(K-1)\pi n/L}. \qquad (7)
\end{aligned}
$$

Both the mean and variance of $\hat{P}_{noise}$ are important performance measures to evaluate the quality of the estimation. Now, let us compute the mean and variance of $\hat{P}_{noise}$.

The mean of $\hat{P}_{noise}$ is computed as

$$
\begin{aligned}
E\left[\hat{P}_{noise}\right] &= \frac{L^2}{L_w K}\sum_{n=(L-L_w)/2}^{(L+L_w)/2-1}\left|\sqrt{P_1}h_p(n)+w_M(n)\right|^2 \qquad (8) \\
&= \frac{P_1}{L_w K}\sum_{n=(L-L_w)/2}^{(L+L_w)/2-1}\left|\frac{\sin(K\pi n/L)}{\sin(\pi n/L)}\right|^2+\sigma^2, \qquad (8)
\end{aligned}
$$

holdsthe power of $w_P(n)$ is evenly distributed over all time indexand is from Parseval'stheorem.

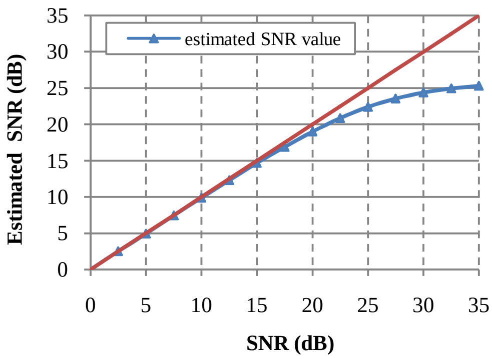

Figure 3. The estimated SNR

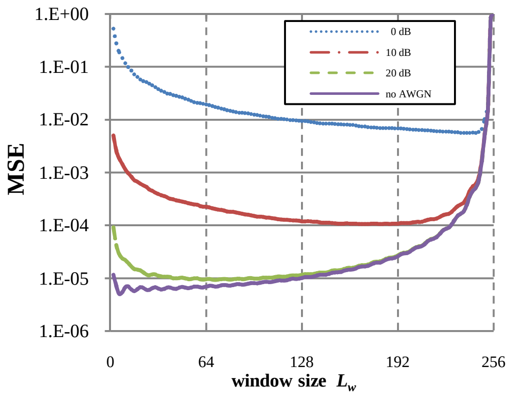

Figure 4. MSE versus window size

From (8), it can be noticed that the estimation is composed of two parts. One part results from the average power of $\sqrt{P_1}h_p(n)$ over a time window of size L_w and the other results from the AWGN power. One part is a bias term for the noise estimation.

The variance of $\hat{P}_{noise}$ is computed as(9) is

$$
\begin{aligned}
Var\left[\hat{P}_{noise}\right] &= Var\left[\frac{L^2}{KL_w}\sum_{n=(L-L_w)/2}^{(L+L_w)/2-1}\left|w_M(n)\right|^2\right] \\
&= \frac{L}{L_w K}\sigma^4, \qquad (9)
\end{aligned}
$$

from $E[\|W_I(k)\|^2] = \sigma^2/2$ andParseval's theorem. The variance hasone result from the AWGN power.Thevariance of the estimation becomes smaller, as we increase the L_w/L value, the ratio between the window size and the IFFT size used for the noise estimation.

The Mean Square Error (MSE) of the noise power estimation is obtained as

$$
\begin{aligned}
MSE(\hat{P}_{noise}) &= E[(\hat{P}_{noise}-\sigma^2)^2] \\
&= \left(\frac{P_1}{L_w K}\sum_{n=(L-L_w)/2}^{(L+L_w)/2-1}\left|\frac{\sin(K\pi n/L)}{\sin(\pi n/L)}\right|^2\right)^2+\frac{L}{L_w K}\sigma^4, \qquad (10)
\end{aligned}
$$

the MSE of the estimation is composed of two terms. The first term is from the bias of the estimation and the second term is from the variance of the measured value. Generally, the first term increases and the second term decreases, as the window size L_w increases. Therefore, it is very important to choose a good window size L_w to obtain a reliable noise estimation value.

4. Results

In this section, results are presented for the proposed the noise power estimation algorithm. To obtain the results, it is assumed that thenumbers of pilots and subcarriers are 200 and1200, the same asa20MHz bandwidth LTE system. The receiver uses an IFFT size of 256 to compute the time domain channel response using the pilots.

Figure 3.shows the estimated SNR from $\hat{P}_{noise}$. The SNR performance degrades as the input SNR increases. The estimated SNR is saturated to about SNR=25 dB because of the bias term.

Figure 4.showsthe mean square error $MSE(\hat{P}_{noise})$ versus window sizeL_w, when the channel is complex AWGN channel.From (10), without the bias term, it is expected that MSE decreases by 20dB, when SNR decreases by 10dB. It can be observed that the MSE difference is about 20 dB between SNR = 0 dB and 10 dB cases. However, between SNR = 10 dB and 20dB cases, the MSE difference is only about 10dB, which results from the bias term in the noise power estimation.

5. Conclusion

In many OFDM communication systems, pilots are periodically transmitted for channel estimation. In this paper, a noise power estimation algorithm is investigated based on the time-domain channel response obtained from the pilot signals. By performance analysis and simulation, the performance of the noise estimation algorithm is evaluated. It is observed that the noise estimation considered in this paper shows good performance at low SNR regimes. However, it has a bias and the estimated SNR saturates to a certain value at high SNR regimes.

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